**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

**Ans:**

*μ* = 45 minutes

*σ* = 8 minutes

We need to find the probability that the manager cannot meet the commitment,

* P(X>50) = 1-P(X<=50)

stats.norm.cdf (50, loc=45, scale=8) , is **73.4%**

* P(X>50) = 1-P(X<=50)

1-stats.norm.cdf (50, loc=45, scale=8) is **26.59%**

**Option B**

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**Ans:**

Population = 400

*μ* = 38

*σ* =6

1. P(X>44) and P(38<X<44)

P(X<=44) =84.13%

* P(X>44) => 1 - Stats.norm.cdf(44, 38, 6) =**15.86%**
* P(38<X<44)=> Stats.norm.cdf(44, 38, 6) – stats.norm.cdf (38,38,6) =**34.13%**

Since, P (38<X<44) > P(X>44), the statement given is False, meaning that the population between age 38 and 44 is more than the population with age greater than 44.

1. P(X<30)

Stats.norm.cdf (30, 38, 6) =**9.12%**

So, number of employees is 0.0912\*400 = 36.48 = approximately 36 employees. Thus, the statement given is TRUE

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**Ans:**

* *X*~ *N*(μ1, σ1^2) and *Y*~ *N*(μ2, σ2^2) are two independent random variables then X+Y~N(μ1+ μ2, σ1^2 + σ2^2) and X-Y~N(μ1- μ2, σ1^2 + σ2^2)
* Similarly if Z = aX + bY , where X and Y are as defined above , i.e Z is linear combination of X and Y , then Z ∼ N(aµ1 + bµ2, a^2σ1^2 + b^2σ2^2 ).
* Therefore in the question 2X1~ N(2 u,4 σ^2) and X1+X2 ~ N(µ + µ, σ^2 + σ^2 ) ~ N(2 u, 2σ^2 ) 2X1-(X1+X2) = N( 4µ,6 σ^2)

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

**Ans:**

X ~ N (100, 202)

Here std = 20 and Mean= 100

Probability of any random variable between a and b is 0.99. So the probability of area outside is 1-0.99 = 0.01

Alpha is 0.01

The probability towards left from a is = 0.01/2 = **-0.005**

The probability towards right from b is = 0.01/2 = **0.005**

Z=(X- μ) / σ

Z score of 0.005 is -2.57

Z \* σ + μ = X

-(-2.57) \* 20 + 100 = **151.4**

(-2.57) \* 20 + 100 = **48.6**

**Option D**

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

Profit1 ~ N(5, 32)

Mean = 5, std = 9

Profit2 ~ N(7, 42)

Mean = 7, std = 16

Total profit and std = profit 1 + profit 2 ~ N[(5+7),(9+16)] = N(12,25)

$1 = Rs. 45

a)

At 95% , z score = stats.norm.ppf(0.975) = 1.96

Z=(X- μ) / σ

Z \* σ + μ = X

Therefore lower rupee range = X= (-1.96\*5 )+12=2.2$ =2.2\*45=99 rupees upper rupee range = X= (1.96\*5 )+12 =21.8 $ =21.8\*45 =981 rupees

b)

5th percentile

To find the 5th percentile value of profit we have to find

Z alpha =Z 0.05 = - 1.644584 , Therefore 5th percentile value of

profit =( -1.644584 \* 5) +12 =3.777 $ =3.777\*45 =170 rupees

c)

1- Profit probability

1 – stats.norm.cdf()

Probability of First company not making any profit, profit = 0: (if there is no profit, then they are in loss)

Not making profit , x = 0

Profit1 ~ N(5, 32) => mean is = 5 and standard deviation = 3^2

Z=(X- μ) / σ

* (0 – 5)/ √9
* -1.667

Profit2 ~ N(7, 42) => mean is = 7 and standard deviation = 4^2

Z=(X- μ) / σ

* (0 – 7)/ √16
* -1.75

Thus the probability that the second company making more loss is second division in the company.